

Constraining Supergravity Scenarios through the $b \rightarrow s, \gamma$ Decay*

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Abstract

We evaluate the branching ratio $\text{BR}(b \rightarrow s, \gamma)$ in the minimal supersymmetric standard model (MSSM), determining the corresponding phenomenological restrictions on two attractive supergravity scenarios, namely minimal supergravity and a class of models with a natural solution to the μ problem. We have included in the calculation some one-loop refinements that have a substantial impact on the results. The numerical results show some disagreements with part of the previous results in the literature, while they are in agreement with others. For minimal supergravity the CLEO upper and lower bounds put important restrictions on the scalar and gaugino masses in both cases $\mu < 0$ and $\mu > 0$. For the other supergravity scenarios the relevant CLEO bound is the upper one. It is stressed the fact that an eventual improvement of the experimental bounds of order 10^{-4} would strengthen the restrictions on the MSSM dramatically. This would be enough to discard these supergravity scenarios with $\mu < 0$ if no discrepancy is found with the standard model prediction, while for $\mu > 0$ there will remain low-energy windows.

January 1995

*Work partly supported by CICYT under contract AEN94-0928, and by the European Union under contract No. CHRX-CT92-0004.

[†]Supported by a Spanish M.E.C. Postdoctoral Fellowship.

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1 Introduction

It is well known that the $b \rightarrow s, \gamma$ process has the potential to put relevant constraints to physics beyond the Standard Model (SM). This comes mainly from two facts. First, as a FCNC process, the $b \rightarrow s, \gamma$ decay occurs only beyond the tree level, thus being sensitive to the characteristics of possible new particles circulating in the relevant loops. Second, there are available experimental data on the exclusive [1] $B \rightarrow K^*, \gamma$ and the inclusive [2] $B \rightarrow X_s, \gamma$ decays that lead to upper and lower bounds on the branching ratio $\text{BR}(b \rightarrow s, \gamma)$ of the same order as the SM prediction. In particular, from the inclusive B decay: $1 \times 10^{-4} < \text{BR}(b \rightarrow s, \gamma) < 4 \times 10^{-4}$. There has recently been a large amount of work discussing the prediction for $\text{BR}(b \rightarrow s, \gamma)$ in the SM and extensions of it, particularly in the Minimal Supersymmetric Standard Model (MSSM) [3, 4, 5, 6, 7, 8]. The latter scenario probably represents the most promising and theoretically well founded extension of the SM capable of giving new measurable physics [9, 10, 11]. In it, besides the usual SM diagram with a W gauge boson and a top quark in the loop, there are additional contributions coming from loops involving charged Higgses (H^-) and a top quark, charginos (χ^-) and u-type squarks (of which the relevant contributions come from the stops, $\tilde{t}_{L,R}$, and scharms, \tilde{c}), and a gluino (χ_3) or neutralinos (χ_i^0) plus a d-type squark (mainly \tilde{b} and \tilde{s}) [3]. As pointed out in ref. [3], the latter two diagrams do not contribute significantly to the BR and can therefore be neglected. So we end up with the following expression for the branching ratio at next to leading order (in units of the BR for the semileptonic b decay):

$$\frac{\text{BR}(b \rightarrow s\gamma)}{\text{BR}(b \rightarrow ce\bar{\nu})} = \frac{|K_{ts}^* K_{tb}|^2}{|K_{cb}|^2} \frac{6\alpha_{QED}}{\pi} \times \frac{\left[\eta^{16/23} A_\gamma + \frac{8}{3}(\eta^{14/23} - \eta^{16/23}) A_g + C\right]^2}{I(z)} F \quad (1)$$

where $z = \frac{m_c}{m_b}$, $\eta = \frac{\alpha_s(M_W)}{\alpha_s(m_b)}$, $I(z) = 1 - 8z^2 + 8z^6 - z^8 - 24z^4 \log(z)$ is the phase space factor, C stands for the leading logarithmic QCD corrections which have been calculated in refs. [12, 13, 14, 15], and $F \sim 1 - \frac{8}{3} \frac{\alpha_s(m_b)}{\pi} \frac{1}{\kappa(z)}$ contains NLO effects ($\kappa(z)$ being the NLO correction to the semileptonic decay)¹. Finally, $A_{\gamma,g}$ are the coefficients of the effective operators for the $bs\gamma$ and bsg interactions; in our case, as mentioned before, we consider as relevant the contributions coming from the SM diagram plus those with top quark and charged Higgs, and stops/scharms and charginos running in the loop. Their expressions are given by:

$$A_{\gamma,g}^{SM} = \frac{3}{2} \frac{m_t^2}{M_W^2} f_{\gamma,g}^{(1)} \left(\frac{m_t^2}{M_W^2} \right)$$

$$A_{\gamma,g}^{H^-} = \frac{1}{2} \frac{m_t^2}{m_H^2} \left[\frac{1}{\tan^2 \beta} f_{\gamma,g}^{(1)} \left(\frac{m_t^2}{m_H^2} \right) + f_{\gamma,g}^{(2)} \left(\frac{m_t^2}{m_H^2} \right) \right]$$

¹Notice that we have chosen $Q = m_b$ as our renormalization scale. For a discussion about the uncertainties derived from this election see ref. [15].

$$\begin{aligned}
A_{\gamma,g}^{\chi^-} = & \sum_{j=1}^2 \left\{ \frac{M_W^2}{M_{\chi_j}^2} \left[|V_{j1}|^2 f_{\gamma,g}^{(1)} \left(\frac{m_{\tilde{c}}^2}{M_{\chi_j}^2} \right) - \sum_{k=1}^2 \left| V_{j1} T_{k1} - \frac{V_{j2} m_t T_{k2}}{\sqrt{2} M_W \sin \beta} \right|^2 \right. \right. \\
& \times \left. \left. f_{\gamma,g}^{(1)} \left(\frac{m_{\tilde{t}_k}^2}{M_{\chi_j}^2} \right) \right] - \frac{U_{j2}}{\sqrt{2} \cos \beta} \frac{M_W}{M_{\chi_j}} \left[V_{j1} f_{\gamma,g}^{(3)} \left(\frac{m_{\tilde{c}}^2}{M_{\chi_j}^2} \right) \right. \right. \\
& \left. \left. - \sum_{k=1}^2 \left(V_{j1} T_{k1} - V_{j2} T_{k2} \frac{m_t}{\sqrt{2} M_W \sin \beta} \right) T_{k1} f_{\gamma,g}^{(3)} \left(\frac{m_{\tilde{t}_k}^2}{M_{\chi_j}^2} \right) \right] \right\} \quad (2)
\end{aligned}$$

where the functions $f_{\gamma,g}^{(i)}$, $i = 1, 2, 3$ were originally calculated in ref. [3], and all the masses are understood to be at the electroweak scale. V and U are the matrices which diagonalise the chargino mass matrix, while T diagonalises the stop mass matrix.

As we can see from the former expressions, it is not easy to draw general conclusions about the behaviour of these amplitudes due to their complicated dependence on the different masses and matrix elements. However, there are some general features which are worth mentioning: for example, both the SM and the charged Higgs contributions have always the same sign, giving therefore a total amplitude which is bigger than the SM one. This fact has been used by several authors [13, 16] to impose severe restrictions on two Higgs doublets models (2HDM); in our case this effect is not as sharp due to the presence of the chargino contribution which, for a wide range of the parameter space, has opposite sign to that of the other two amplitudes. As we will see this is not enough, in general, to lower the total BR below the SM prediction, although in some cases it might be possible to have such big values for $A_{\gamma,g}^{\chi^-}$ as to drive the total BR even below the CLEO lower bound.

In the formulation of the MSSM the mass matrices of charginos, charged Higgses, stops, etc, appearing in the above expressions, are not independent parameters. They come as a low energy result of the actual initial parameters of the theory, which are assumed to be given at the unification scale, M_X . It is therefore of great importance to correctly establish the constraints on these parameters from $b \rightarrow s, \gamma$ measurements. Several recent papers have been concerned with this task [4, 5, 6, 7, 8]. However, as we will see, the complete one-loop effects to be taken under consideration have not been always properly included, particularly in the works dealing with the minimal supergravity (SUGRA) scenario. A careful inclusion of these effects, especially the radiative electroweak breaking, is precisely the main goal of this article.

In addition, most of the SUGRA analysis have just dealt with the minimal SUGRA scenario, which is only justified by its simplicity. In fact, this scenario has a naturalness problem, the so-called μ -problem, namely the origin of the electroweak size of the μ parameter in the superpotential (see below). In recent times, however, there have appeared string and grand unification well motivated mechanisms to solve the μ -problem [17, 18, 19, 20, 21] that imply a departure from minimal SUGRA. It is also the aim of this article to study the constraints on these scenarios coming from the $b \rightarrow s, \gamma$ process.

In section 2 we introduce two supergravity scenarios, labelled SUGRA I and SUGRA II,

which correspond to the previously mentioned supergravity frameworks, and establish their connection to the MSSM. In section 3 we explain our approach to determine the restrictions on the MSSM coming from the $b \rightarrow s, \gamma$ decay, focussing our attention on some one-loop refinements that have a substantial impact on the results. The numerical results for the supergravity scenarios under consideration and their implications are presented and discussed in section 4. Finally, we present our conclusions in section 5.

2 MSSM and SUGRA scenarios

The MSSM is defined by the superpotential, W , and the form of the soft supersymmetry breaking terms. W is given by

$$W = \sum_i \{h_{u_i} Q_i H_2 u_i^c + h_{d_i} Q_i H_1 d_i^c + h_{e_i} L_i H_1 e_i^c\} + \mu H_1 H_2 \quad , \quad (3)$$

where i is a generation index, Q_i (L_i) are the scalar partners of the quark (lepton) SU(2) doublets, u_i^c, d_i^c (e_i^c) are the quark (lepton) singlets and $H_{1,2}$ are the two SUSY Higgs doublets; the h -factors are the Yukawa couplings and μ is the usual Higgs mixing parameter. In all terms of eq. (3) the usual SU(2) contraction is assumed, e.g. $\mu \epsilon_{ij} H_1^i H_2^j$ with $\epsilon_{12} = -\epsilon_{21} = 1$. From W the (global) supersymmetric part of the Lagrangian is readily obtained

$$\mathcal{L}_{SUSY} = - \sum_j \left| \frac{\partial W}{\partial \phi_j} \right|^2 - \frac{1}{2} \left(\sum_{j,k} \left[\frac{\partial^2 W}{\partial \phi_j \partial \phi_k} \right] + \text{h.c.} \right) + \text{D-terms} \quad , \quad (4)$$

where $\phi_{j,k}$ ($\psi_{j,k}$) run over all the scalar (fermionic) components of the chiral superfields. In addition to this, the soft breaking terms coming from the (unknown) supersymmetry breaking mechanism have the form

$$\begin{aligned} \mathcal{L}_{\text{soft}} = & \frac{1}{2} M \lambda_a \lambda_a - \left\{ \sum_j m^2 |\phi_j|^2 + \sum_i A [h_{u_i} Q_i H_2 u_i^c + h_{d_i} Q_i H_1 d_i^c \right. \\ & \left. + h_{e_i} L_i H_1 e_i^c + \text{h.c.}] + [B \mu H_1 H_2 + \text{h.c.}] \right\} \quad , \quad (5) \end{aligned}$$

where a is gauge group index, λ_a are the gauginos, and the remaining fields in the formula denote just the corresponding scalar components. The parameters m , M , A , B are the universal scalar and gaugino masses and the universal coefficients of the trilinear and bilinear scalar terms, respectively. The universality is assumed at the unification scale M_X .

With the previous definitions the chargino mass term in matrix form, which plays a crucial role in the expresions for $\text{BR}(b \rightarrow s, \gamma)$ [see eqs. (1, 2)], is given by

$$- (-i\lambda^-, \tilde{H}_1^-) \begin{pmatrix} M_2 & \sqrt{2} M_W \sin \beta \\ \sqrt{2} M_W \cos \beta & -\mu \end{pmatrix} \begin{pmatrix} -i\lambda^+ \\ \tilde{H}_2^+ \end{pmatrix} \quad , \quad (6)$$

where M_2 is the (renormalized) SU(2) gaugino mass, $\lambda^\pm = (\lambda_1 \mp i\lambda_2)/\sqrt{2}$ and the tildes denote fermionic components.

As has been noted in ref. [6], the sign of μ in the chargino mass matrix has been incorrectly written in ref. [22]; this is also the case for the other diagonal entry (M_2). The same mistake with the μ sign occurs in ref. [11]. On the other hand, the signs of the chargino mass matrix have been consistently written in refs. [9, 23, 7, 6].

The particular values of the soft breaking parameters m, M, A, B depend on the type of SUGRA theory from which the MSSM derives and on the mechanism of supersymmetry breaking. We will consider two types of SUGRA theories without specifying the mechanism of supersymmetry breaking.

SUGRA I

This is just the minimal supergravity theory. It is defined by a Kähler potential $K = \sum_j |\phi_j|^2$ and a gauge kinetic function $f_{ab} = \delta_{ab}$, so that all the kinetic terms are canonical, whereas the superpotential W is assumed to be as in eq. (3), μ being an initial parameter. Then, irrespectively of the supersymmetry breaking mechanism, the gaugino and scalar masses are automatically universal and the coefficients A and B are universal and related to each other by the well known relation

$$B = A - m \quad . \quad (7)$$

This supergravity theory is attractive for its simplicity and for the natural explanation that offers to the universality of the soft breaking terms. Actually, universality is a desirable property not only to reduce the number of independent parameters, but also for phenomenological reasons, particularly to avoid unwanted FCNC and CP violating effects (see e.g. ref. [10]). However, this scenario has a serious drawback, namely the unnaturally small (electroweak) size of the initial μ parameter in the superpotential. This is the so-called μ problem, that leads us to the next supergravity scenario.

SUGRA II

Recently, there have appeared several attractive mechanisms to solve the μ problem that, quite remarkably, lead to a similar prediction for the value of B . Very briefly, these are the following:

- a) In ref. [18] was noted that if the superpotential has a non-renormalizable term of the form

$$\lambda W_o H_1 H_2 \quad , \quad (8)$$

where W_o is the renormalizable superpotential and λ an unknown coupling, then a μ term is automatically generated with size $\mu = \lambda m_{3/2}$. The B parameter can also be straightforwardly evaluated. Assuming that the Kähler potential is as in minimal supergravity in order to guarantee universal soft breaking terms, the simple result is

$$B = 2m \quad . \quad (9)$$

For this mechanism to work, μ must be vanishing at the renormalizable level, a fact that, remarkably enough, is automatically guaranteed in the framework of string theories [18]. On the other hand, the existence of a non-renormalizable term as that of eq. (8) is also quite natural in superstring theories (see e.g. ref. [24]). This is equivalent for practical purposes to the presence of a term $\lambda H_1 H_2 + \text{h.c.}$ in the Kähler potential, which resembles very much the mechanism proposed in ref. [17] to solve the μ problem, namely the presence of a term like $\lambda Z^* H_1 H_2$ in K , where Z^* is a “hidden” field acquiring a large VEV.

- b) In ref. [19] it was made the observation that in the framework of any SUSY–GUT theory, starting again with $\mu = 0$, an effective μ -term is generated by the integration of the heavy degrees of freedom. Assuming again a scenario with minimal Kähler metric, so that the kinetic terms are canonical and the soft breaking terms universal, the prediction for B is once more $B = 2m$, as in eq. (9).
- c) Finally, let us note that the assumption of minimal Kähler potential, K , is not really justified from string theories. The form of K in a large class of phenomenologically interesting constructions was obtained in ref. [25] and the corresponding soft breaking terms in refs. [26, 27]. It was noted, however, that the assumption of universality can still be quite reasonable (apart from phenomenologically desirable). It occurs e.g. in the so-called dilaton–dominance limit [20, 21]. Concerning the μ problem and the value of B , it was noted in ref. [21] that in the case of dilaton–dominance and a μ -term arising as suggested in ref. [17] or in ref. [18] with $\lambda = \text{const.}$ (see eq. (8)), then once again the value $B = 2m$ is obtained for large-size Calabi–Yau manifolds and orbifold compactifications. In the latter case the two Higgses must belong to the untwisted sector or possess similar modular weights.

It is not clear by now what is the reason behind the coincidence in the prediction for B in the previous three different supergravity scenarios. Probably, it is a more general fact than just a characteristic of these scenarios. In any case, we will refer (any of) them as SUGRA II. The corresponding MSSM emerging from them is as in minimal supergravity except for the value of B , which is given by (9) instead of (7).

3 Our calculation

The aim of our calculation is to obtain the restrictions on the MSSM from the $b \rightarrow s, \gamma$ decay in the two supergravity scenarios, SUGRA I and SUGRA II, above defined. The scheme of our procedure is the following. We start with the MSSM with initial parameters

$$\alpha_X, M_X, h_t, h_b, h_\tau, \mu, m, M, A, B \quad (10)$$

By consideration of one of the two previous SUGRA theories we eliminate the B parameter through eq. (7) or eq. (9). Then we demand consistency with all the experimental data (apart from $b \rightarrow s, \gamma$). More precisely, we require

- i*) Correct unification of the gauge coupling constants².
- ii*) Correct masses for all the observed particles.
- iii*) Masses for the unobserved particles compatible with the experimental bounds.
- iv*) Correct electroweak breaking, i.e. $M_Z = M_Z^{exp}$.

Conditions *(i)–(iii)* allow to eliminate $\alpha_X, M_X, h_t, h_b, h_\tau$, while *(iv)* allows to eliminate one of the remaining parameters, which we choose to be μ . This finally leaves three independent parameters, namely

$$m, M, A \quad . \quad (11)$$

Actually, for many choices of these parameters there is no value of μ capable of producing the correct electroweak breaking. Therefore, this requirement restricts the parameter space substantially. On the other hand, there can be two branches of solutions depending on the sign of μ . Finally, we evaluate $\text{BR}(b \rightarrow s, \gamma)$ and compare it with the experimental measurements, eventually obtaining the restrictions on the initial parameters of theory coming from the $b \rightarrow s, \gamma$ decay. We will also express the restrictions in terms of some significative low-energy parameters, in particular $\tan \beta$.

In order to be consistent, there is a number of refinements that have to be considered in the calculation. Before explaining them, let us stress that their importance is enhanced by the following fact. It is known that in order to get a correct electroweak breaking in the context of the MSSM a certain amount of fine-tuning between the initial parameters is normally necessary [28]. It is a matter of opinion what is the maximal acceptable amount of fine-tuning and how to measure it [28, 29, 30, 31]. But, in any case, this means that rather small variations on the initial parameters may have important implications at low energy. Therefore, the determination of what are the initial parameters compatible with the experimental conditions *(i)–(iv)* has to be made in a very careful way.

The first refinement concerns condition *(i)*. Precision measurements at LEP give a strong support [32] to the expectation of supersymmetric unification of the gauge coupling constants³ at $M_X \sim 10^{16}$ GeV, $\alpha_X \sim 0.04$. Actually, the unification can only be achieved when the renormalization group equations (RGE) of the gauge couplings are taken at two-loop order. For consistency, all the supersymmetric thresholds (and the top quark one) have to be taken into account in the running in a separate way.

²Strictly speaking, this is not an experimental observation, but it is a fact strongly suggested by the data that nicely fits with the theoretical expectatives. We will turn to this point below.

³This unification does not necessarily require a GUT. In particular, in superstring theories all the gauge couplings are essentially the same at tree level [33], even in the absence of a grand unification group.

This amounts to a technical problem since the values of the supersymmetric masses depend themselves on the initial parameters of the theory at M_X . Therefore, for a given model, i.e. for a choice of the initial parameters, one cannot know a priori whether the resulting values of $\alpha_1(M_Z), \alpha_2(M_Z), \alpha_3(M_Z)$ will be consistent with their experimental values, namely [32, 34]

$$\begin{aligned}\alpha_1(M_Z) &= 0.016887 \pm 0.000040 \\ \alpha_2(M_Z) &= 0.03322 \pm 0.00025 \\ \alpha_3(M_Z) &= 0.124 \pm 0.006\end{aligned}\tag{12}$$

This means that in order to include the perturbative unification in a consistent way an iterative process is necessary to adjust the initial parameters so that (12) is fulfilled.

A second refinement has to do with the masses of the fermions. The Yukawa couplings of the top and bottom quarks and the tau lepton, h_t, h_b, h_τ , play an important role in the radiative electroweak breaking. Their boundary conditions have to be chosen so that the experimental masses are fitted according to condition (ii). However, the running masses defined as $m_t(Q) = \langle H_2 \rangle h_t(Q)$, $m_b(Q) = \langle H_1 \rangle h_b(Q)$, $m_\tau(Q) = \langle H_1 \rangle h_\tau(Q)$ do not coincide with the physical (pole) masses M_t, M_b, M_τ . In particular, for the top quark [35]

$$M_t = \left\{ 1 + \frac{4}{3} \frac{\alpha_S(M_t)}{\pi} + \left[16.11 - 1.04 \sum_{i=1}^5 \left(1 - \frac{M_i}{M_t} \right) \right] \left(\frac{\alpha_S(M_t)}{\pi} \right)^2 \right\} m_t(M_t). \tag{13}$$

where M_i , $i = 1, \dots, 5$ represent the masses of the five lighter quarks, and similar expressions for the other quarks. In our calculation we have used the recent evidence for top quark production at CDF with a mass $M_t = 174 \pm 17$ GeV [36], taking the central value. Notice, however, that the masses in the $\text{BR}(b \rightarrow s, \gamma)$ expressions, eqs. (2), are the running masses at the electroweak scale. Incidentally, it is amusing to realize that $m_t(M_Z)$ is extremely close to the pole mass M_t (while this does not happen for the other particles).

Our third refinement, and the most important one, has to do with the electroweak breaking process. The vacuum expectation values (VEVs) of the two Higgses, $v_1 = \langle H_1 \rangle$, $v_2 = \langle H_2 \rangle$ are to be obtained from the minimization of the Higgs potential. The tree level part of this in the MSSM has the form

$$V_o = m_1^2 |H_1|^2 + m_2^2 |H_2|^2 + 2\mu B H_1 H_2 + \frac{1}{8}(g^2 + g'^2)(|H_2|^2 - |H_1|^2)^2, \tag{14}$$

where all the parameters are understood to be running parameters evaluated at the renormalization scale Q . By a suitable redefinition of the phases of the fields it is always possible to impose $v_1, v_2 > 0$. As was clarified by Gamberini et al. [37], V_o and the corresponding tree level VEVs $v_1^o(Q), v_2^o(Q)$ are strongly Q -dependent. In order to get a much more scale independent potential the one-loop correction ΔV_1 is needed. This is given by

$$\Delta V_1 = \sum_j \frac{n_j}{64\pi^2} M_j^4 \left[\log \frac{M_j^2}{Q^2} - \frac{3}{2} \right], \tag{15}$$

where $M_j^2(\phi, t)$ are the tree-level (field-dependent) mass eigenstates and n_j are spin factors. In this way, the minimization of $V = V_o + \Delta V_1$ gives one-loop VEVs $v_1(Q), v_2(Q)$ much more stable against variations of the Q scale. In general, there is a region of Q where $v_1(Q), v_2(Q)$ are remarkably Q -stable and a particular scale, \hat{Q} , always belonging to that region, at which $v_1(\hat{Q}), v_2(\hat{Q})$ essentially coincide with $v_1^o(\hat{Q}), v_2^o(\hat{Q})$. This is illustrated in Fig. 1 with a particular example. The fact that \hat{Q} always belong to the stability region is not surprising since at \hat{Q} the one-loop correction ΔV_1 is necessarily small, which is the right criterion to choose an appropriate renormalization scale that minimizes the size of the potentially large logarithms, thus optimizing the perturbative expansion. \hat{Q} is a certain average of the masses contributing to ΔV_1 in eq. (15). When we move far from \hat{Q} the logarithms become large and the perturbative expansion breaks down. As a result, $v_1(Q), v_2(Q)$ become strongly Q -dependent and eventually meaningless. Consequently, when \hat{Q} is substantially larger than M_Z (and this is the typical case), it is a bad approximation to minimize $V = V_o + \Delta V_1$ at $Q = M_Z$ as is very commonly done. This is again illustrated in Fig. 1. In our calculation we have evaluated v_1, v_2 at \hat{Q} and then obtained $v_1(Q), v_2(Q)$ at any scale by using the renormalization group running of the H_1, H_2 wave functions. This is necessary in order to get the various masses appearing in the problem at the appropriate scale. Furthermore, we have included in eq. (15) *all* the supersymmetric spectrum since, as was stressed in [29], some common approximations, such as considering only the top and stop contributions, can lead to wrong results.

The previous refinements imply a whole iterative process in order to determine a particular set of sensible initial parameter [eq. (10)]. This is in fact the most painful task of the entire calculation. Then, for each model it is quite straightforward to obtain the mass matrices of the supersymmetric particles and thus the mass eigenvalues, diagonalization matrices, etc, to be plugged in the expressions for the branching ratio, eqs. (2). The results and their implications are presented and discussed in the next section.

4 Results

We will consider the cases SUGRA I and SUGRA II in a separate way.

SUGRA I

As discussed in the previous section, the consideration of the physical requirements (i)–(iv) in the supergravity theory at hand restricts the number of independent parameters to the set m, M, A [see eq. (11)]. In order to present the results in a comprehensible way, let us make for the moment the assumption $m = M$. The corresponding plots of the branching ratio $\text{BR}(b \rightarrow s, \gamma)$ (in short BR) versus the remaining parameter, A/m , for different values of m are given in Fig. 2a (branch $\mu > 0$) and Fig. 2b (branch $\mu < 0$). The lowest represented values of m correspond to the current experimental lower limits on supersymmetric particles [38]. The SM prediction for the BR and the

CLEO upper and lower bounds are also shown. In both figures we observe that the MSSM result smoothly approaches the SM one as m becomes larger, though the limit is achieved quite slowly. For $\mu > 0$ this trend is only observed for $m \geq 200$ GeV, since for $m \leq 200$ GeV the BR behaves in a much more “unpredictable” way.

It is clear from the figures that the present CLEO bounds exclude an appreciable region of the parameter space in both cases $\mu > 0$ and $\mu < 0$. It is interesting to trade the high energy parameter A by the low energy parameter $\tan\beta = v_2/v_1$ in the representation. In this way we obtain the Figs. 3a, 3b. The lowest value of $\tan\beta$ in the two figures is $\tan\beta = 2$, which essentially corresponds to the top infrared-fixed-point value⁴. We can see from the figures that for $m \geq 200$ GeV and both $\mu > 0$, $\mu < 0$, for each value of m there is a maximum acceptable value for $\tan\beta$, e.g. for $m = 300$ GeV we obtain $\tan\beta|_{max} = 12$ ($\mu > 0$), 17 ($\mu < 0$). In general for $m \geq 200$ GeV the restrictions are stronger for positive values of μ and small values of m . For $m \leq 200$ GeV, however, the situation is different: whereas for $\mu < 0$ the restrictions become very strong (only a narrow range of $\tan\beta$ is allowed), for $\mu > 0$ there appear large windows of allowed values of $\tan\beta$. Here the CLEO lower bound plays a relevant role.

The pattern of the restrictions is perhaps better appreciated by plotting the branching ratio vs. m for different values of $\tan\beta$, as is done in Figs. 4a, 4b. For $\mu > 0$, Fig. 4a, we see that for small values of m , there is an allowed window that becomes wider as $\tan\beta$ decreases. For large values of m the result approaches the SM one, eventually becoming compatible with the experimental bounds. For $\tan\beta < 3$ the whole range of m is allowed, a fact that can change as soon as the experimental bounds become a bit better, especially the upper one. For $\mu < 0$, Fig. 4b, we see that the BR behaves in a much more monotonic way. In particular, there are no low-energy windows and there is a one-to-one relation between the minimum acceptable values of m and $\tan\beta$. It is clear from the figures that the trend to the SM result is so slow that an improvement of the CLEO bounds (particularly the upper one) on BR of order 10^{-4} would push the lower limits on m to the TeV region (except for the above-mentioned windows in the $\mu > 0$ case), conflicting with the fine-tuning [28, 29] and cosmological [31] constraints. Strictly speaking the previous results apply only to the $m = M$ case, but their sensitivity to departures from this assumption is rather small, as illustrated in Fig. 5.

We would like at this point to compare the so far presented results with some of the pre-existing ones in the literature [5, 6, 7, 8]. It is in fact worth-noticing that the results of some of these references are rather incompatible each other (see e.g. refs. [5, 8]) although the comparison is in general very difficult to perform. This comes from the fact that the parameters used to exhibit the dependence of the branching ratio are very assorted (usually they are low energy parameters). On the other hand, in some papers it is not specified in the plots what are the values of the non-plotted parameters. In any case, it is possible to detect some disagreements between our results

⁴For $M_{top} = 174$ GeV and h_t with the infrared fixed point boundary condition, $\tan\beta \sim 1.9$. Lower values of $\tan\beta$ cannot be achieved within the MSSM with universal soft terms.

and those of ref. [5] and, especially, ref. [8]⁵. In the latter it is claimed that for $\mu > 0$ (with our convention of signs) the $b \rightarrow s, \gamma$ constraints are dramatically strong, while for $\mu < 0$ they are almost irrelevant. Although we find a trend in this direction, our constraints are not that extreme in any of both senses (see Figs. 2-4). This can also be applied to the results of ref. [6], though the comparison is difficult for the above mentioned reasons. Finally, ref. [7] deals with the infrared fixed point scenario, which in our case corresponds to $\tan\beta \simeq 2$. Here we find a good agreement between our results and theirs.

SUGRA II

We present the results for the SUGRA II scenario in a way completely analogous to those of SUGRA I. In particular, Figs. 6, 7, 8 correspond to Figs. 2, 3, 4 respectively. We note here that the sign of μ is always negative since (adopting the convention $v_1, v_2 > 0$) for $\mu > 0$ and $B = 2m$ the necessary electroweak breaking cannot be achieved. The SUGRA II results present a similar pattern to those of SUGRA I with $\mu < 0$. Again, we find that for a given value of m there is a maximum acceptable value for $\tan\beta$, e.g. for $m = 300$ GeV, $\tan\beta|_{max} = 12$. The bound becomes less stringent as m increases. As in the SUGRA I scenario, the SM limit is very smoothly achieved as m grows, as can be seen from Fig. 8. Therefore, once again, an improvement of the CLEO bounds (especially the upper one) of order 10^{-4} would amount to a dramatical improvement of the MSSM constraints, pushing the limits on m to the TeV region and thus conflicting with fine-tuning and cosmological constraints.

5 Conclusions

We have evaluated the branching ratio $\text{BR}(b \rightarrow s, \gamma)$ in the MSSM that emerges from two attractive supergravity scenarios, SUGRA I and SUGRA II defined in sect. 2, determining the corresponding phenomenological restrictions on the independent parameters of the theory. For consistency, we have included in the calculation some one-loop refinements that have a substantial impact on the results. Our numerical results show some disagreements with part of the previous results in the literature, while they are in agreement with others. For SUGRA I (minimal supergravity) we find that the CLEO upper and lower bounds are capable to put efficient restrictions on m, M (the universal scalar and gaugino masses) in both cases $\mu < 0$ and $\mu > 0$, though the latter tends to be more restrictive. For the SUGRA II scenario the results are similar, but here the relevant role is played by the CLEO upper bound. Since for both scenarios the trend to the SM limit as m increases is very slow, an eventual improvement of the CLEO upper and lower bounds of order 10^{-4} would strengthen dramatically the previously found constraints. This would be enough to discard the SUGRA I with $\mu < 0$ and SUGRA II scenarios if no discrepancy is found with the SM

⁵The origin of these discrepancies is probably due to the refinements explained in sect. 3. One of us (B.C.) is indebted to M.A. Díaz for his help to show that this is essentially the case in ref. [5].

prediction, while for SUGRA I with $\mu > 0$ there will remain low-energy windows.

Acknowledgements

We thank L. Ibáñez and J. Moreno for very useful discussions. We also thank P. Bourdon, O. Diego and J.R. Espinosa for their invaluable help with the computer.

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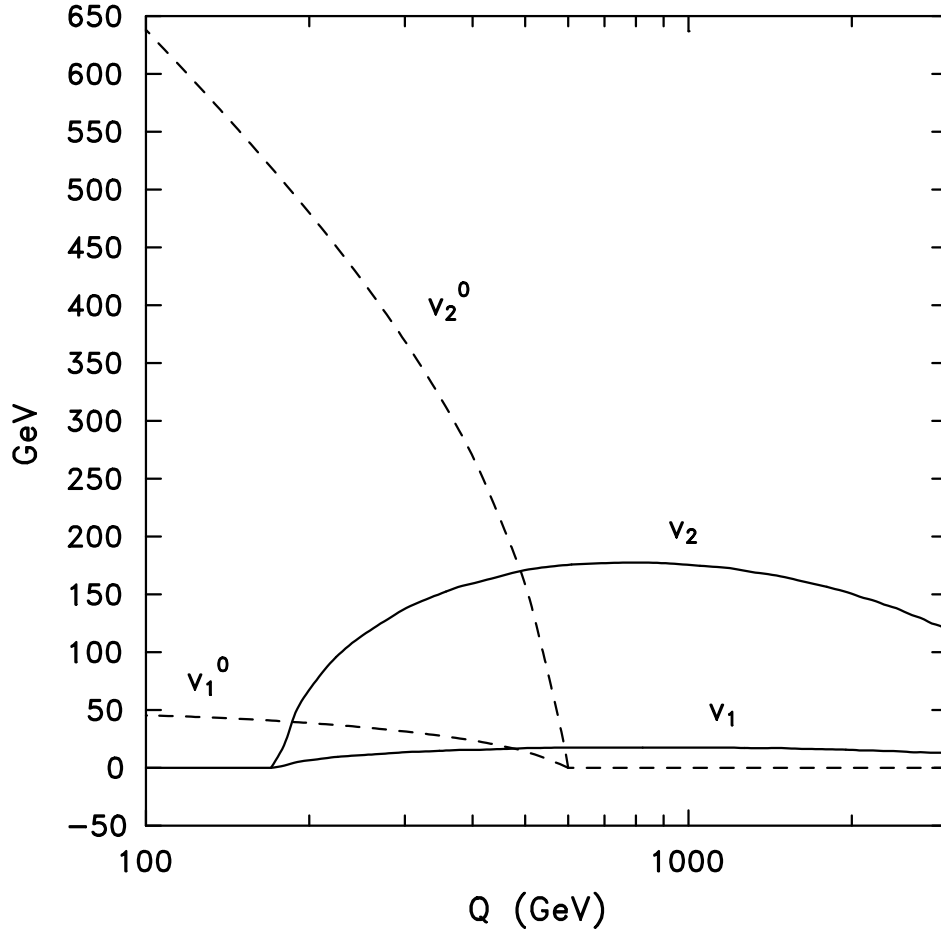


Figure 1: Plot of $v_1 \equiv \langle H_1 \rangle$, $v_2 \equiv \langle H_2 \rangle$ versus the Q scale between 100 GeV and 3 TeV for the supersymmetric model defined by $m = M = 300$ GeV, $A = 317$ GeV, $B = A - m$, $\mu = 403.76$ GeV, $h_t = 0.568$, $h_b = 0.063$, $h_\tau = 0.072$, $\alpha_X = 0.0404$ (all quantities defined at $M_X = 1.60 \times 10^{16}$ GeV). Solid lines: complete one-loop results, dashed lines: (renormalization improved) tree level results.

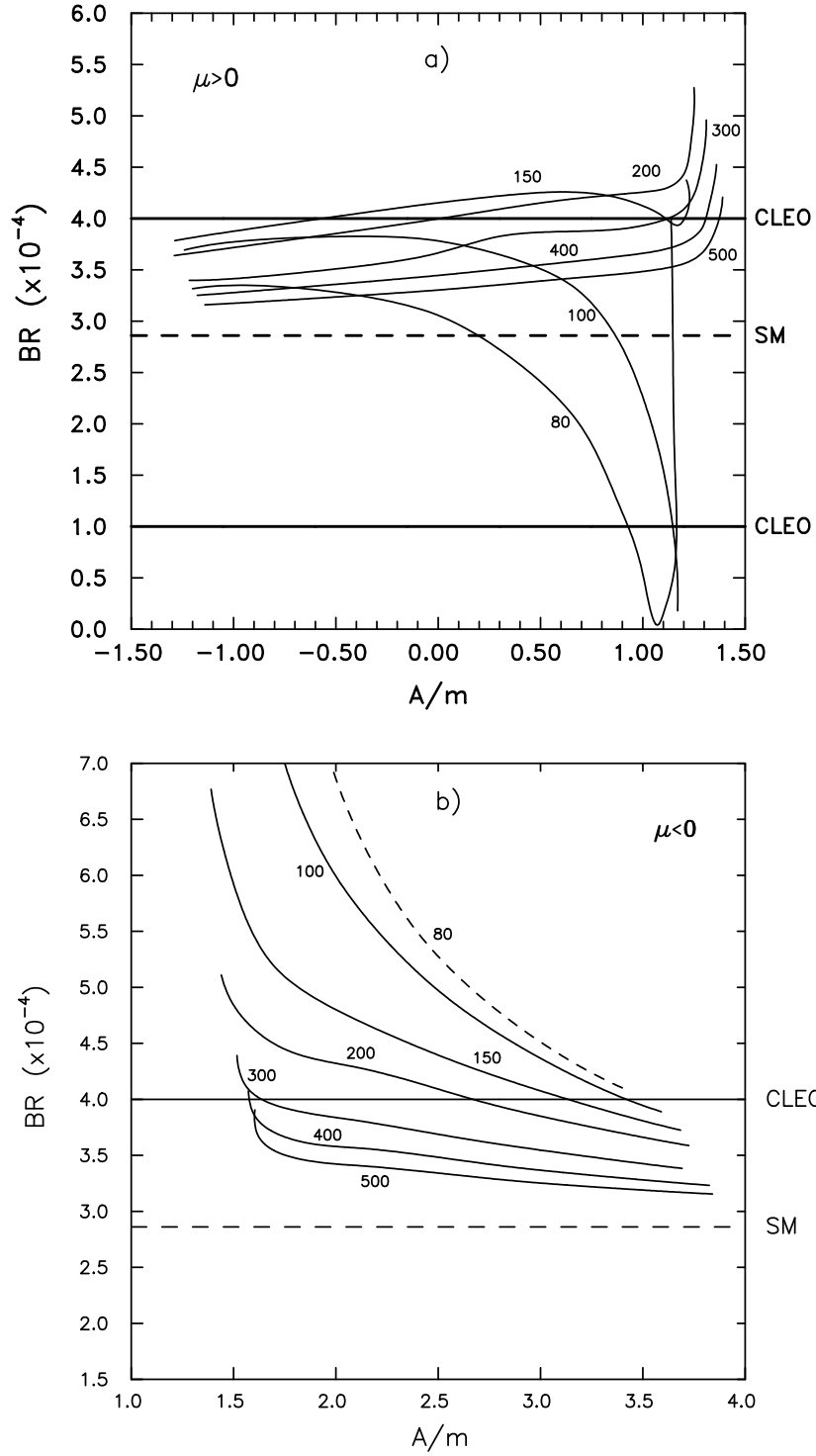


Figure 2: Plot of the branching ratio $BR(b \rightarrow s, \gamma)$ (denoted BR) versus A/m for the SUGRA I scenario (minimal supergravity) with $m = M$ for different values of m (namely, $m = 80, 100, 150, 200, 300, 400, 500$ GeV) : a) branch $\mu > 0$; b) branch $\mu < 0$. The dashed curves indicate that the model becomes incompatible with the experimental lower bounds on supersymmetric particles. The Standard Model prediction (SM) and the CLEO bounds are also shown in the figure.

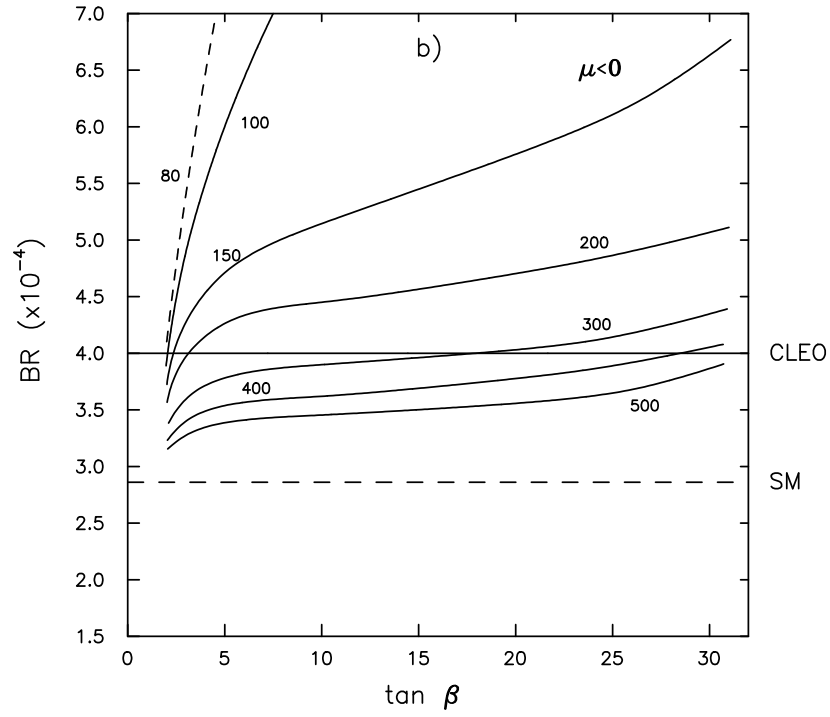
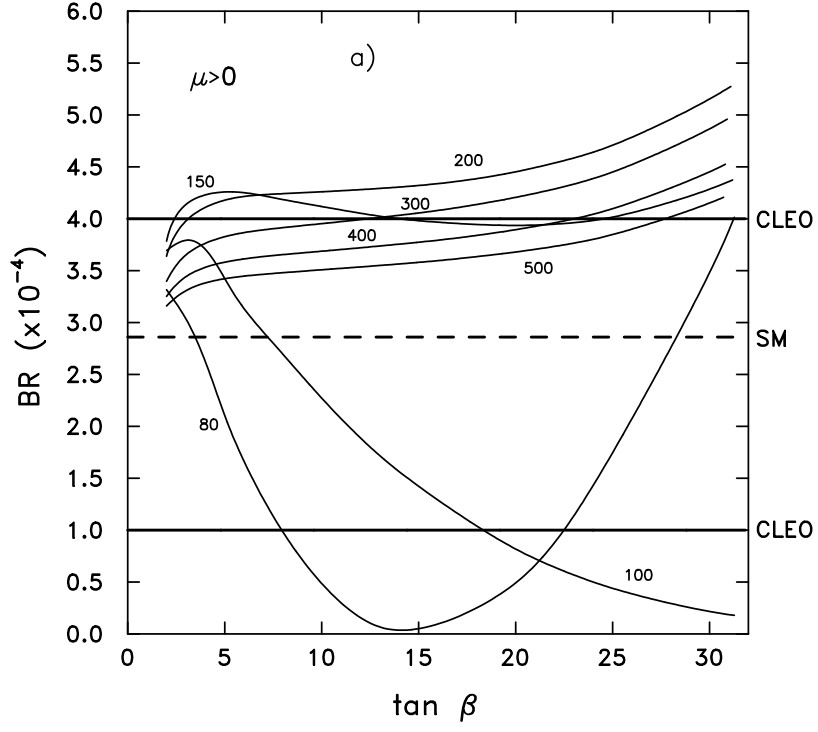


Figure 3: Plot of the branching ratio $BR(b \rightarrow s, \gamma)$ versus $\tan \beta$ for the same models and with the same conventions as in Fig. 2.

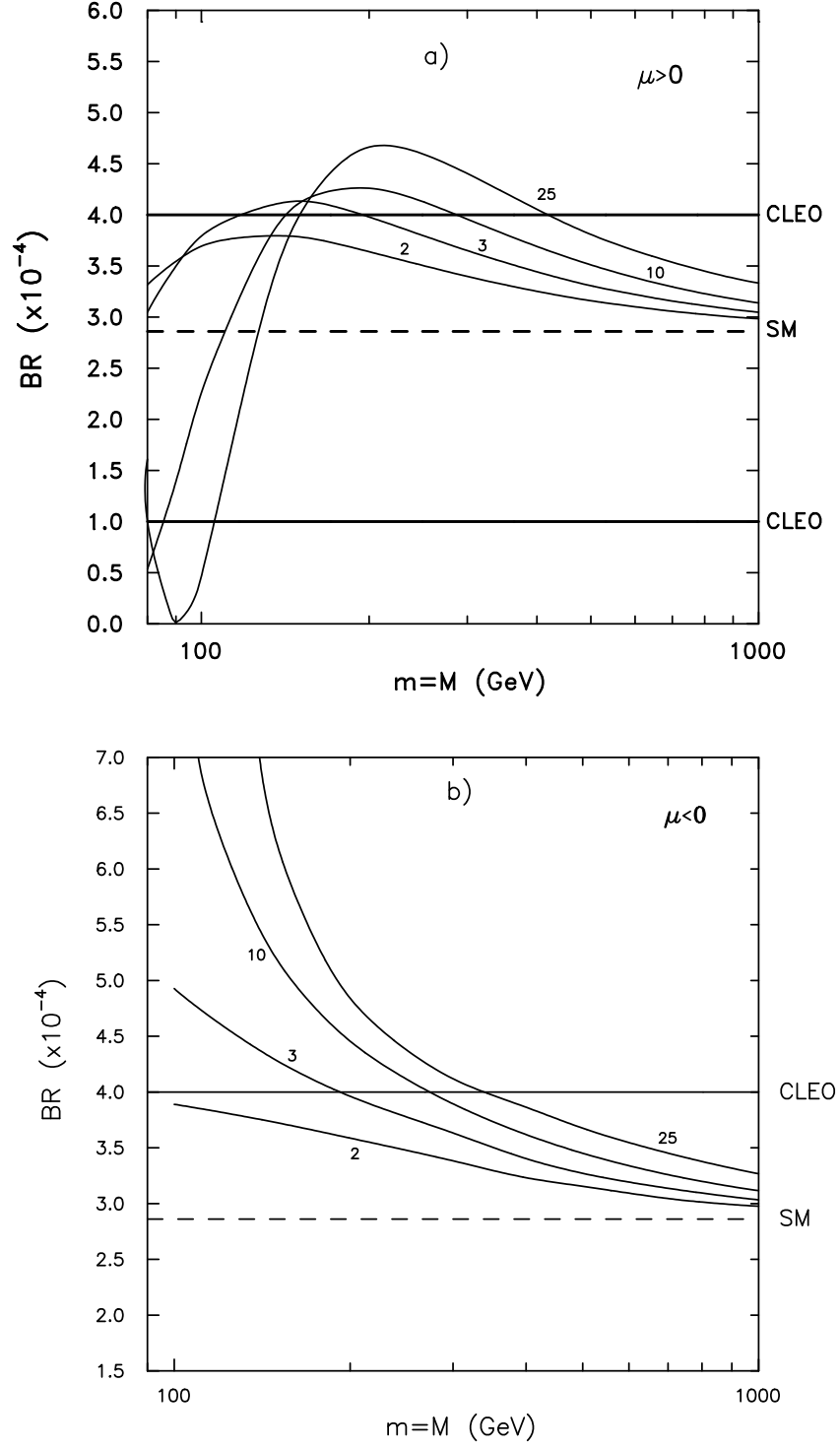


Figure 4: Plot of the branching ratio $BR(b \rightarrow s, \gamma)$ versus $m = M$ for different values of $\tan \beta$ (namely, $\tan \beta = 2, 3, 10, 25$) in the SUGRA I scenario: a) branch $\mu > 0$; b) branch $\mu < 0$.

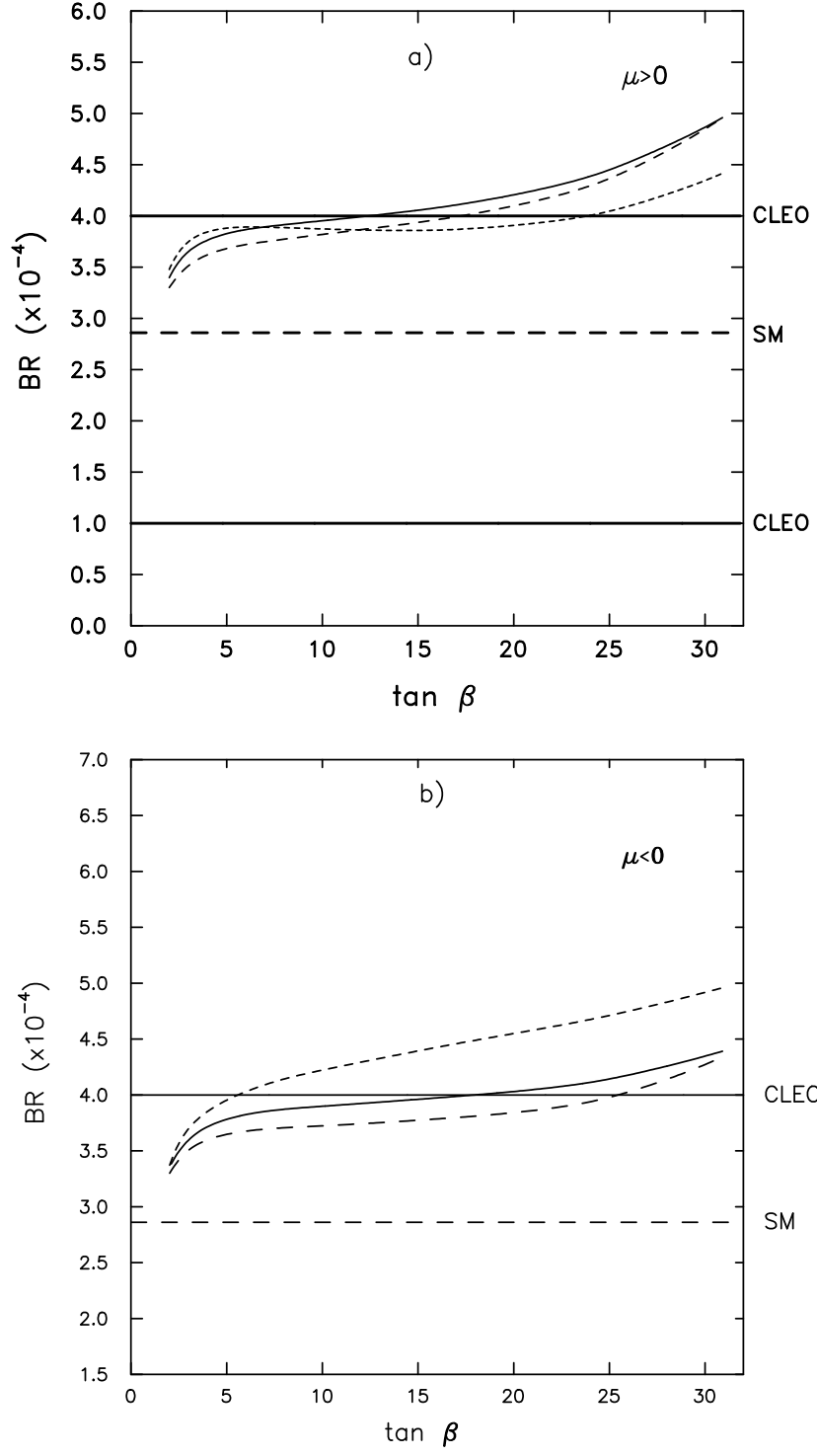


Figure 5: The same as in Fig. 3, but for $m = 300$ GeV and three different values of M . Solid line: $M = 300$ GeV, long-dashed line: $M = 400$ GeV, short-dashed line: $M = 200$ GeV.

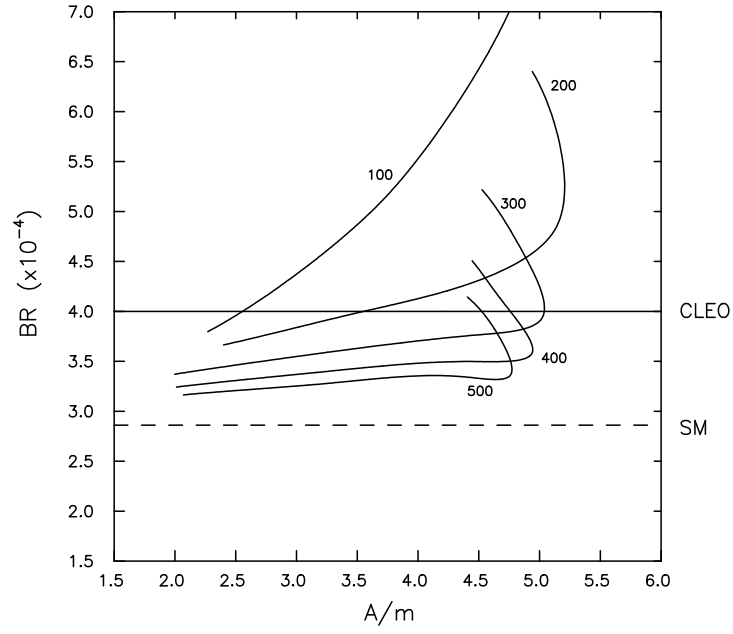


Figure 6: The same as Fig. 2, but for the SUGRA II scenario (see definition in the text), where only the $\mu < 0$ branch is meaningful.

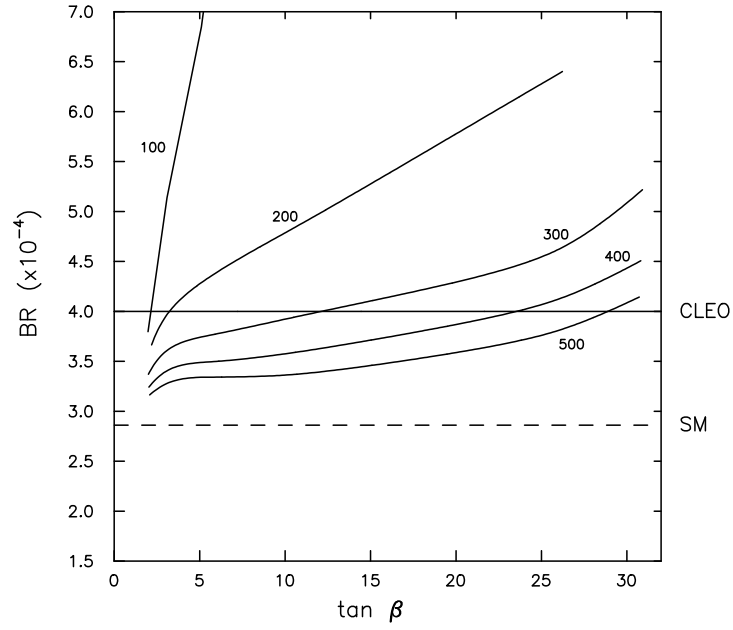


Figure 7: The same as Fig. 3, but for the SUGRA II scenario.

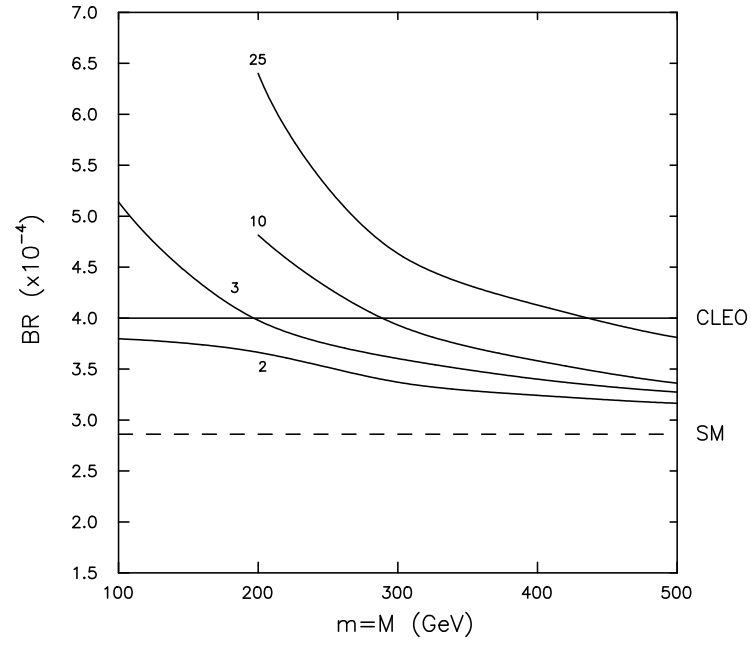


Figure 8: The same as Fig. 4, but for the SUGRA II scenario. The $\tan \beta = 10, 25$ curves have been cut where they start to conflict with the experimental lower limits on the supersymmetric particles.